Naval Research Laboratory

Washington, DC 20375-5320

UNCLASSIFIED

NRL/MR/6707--96-7891

Simple Theory of the Input Coupler to the 94 GHz NRL Gyroklystron

WALLACE M. MANHEIMER

Senior Scientist Fundamental Plasma Processes
Plasma Physics Division

October 31, 1996

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave Blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	October 31, 1996	Interim Report

4. TITLE AND SUBTITLE

Simple Theory of the Input Coupler to the 94 GHz NRL Gyroklystron

5. FUNDING NUMBERS

PE - No. 73

6. AUTHOR(S)

Wallace M. Manheimer

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Research Laboratory
Washington, DC 20375-5320

8. PERFORMING ORGANIZATION REPORT NUMBER

NRL/MR/6707--96-7891

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Office of Naval Research
Arlington, VA 22217-5660

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This Memorandum Report formulates simple approaches to the coupling problem in the NRL 94 GHz gyroklystron. It looks at the coupling holes between the coaxial cavity and main cavity as simple dipoles. It looks at the coupling between the input waveguide and coaxial cavity as either a dipole, or a waveguide T.

14. SUBJECT TERMS

Gyroklystron input coupler
Gyro amplifier
Gyroklystron

15. NUMBER OF PAGES

27

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std 239-18
298-102

CONTENTS

SIMPLE THEORY OF THE INPUT COUPLER TO THE 94 GHz NRL GYROKLYSTRON

1. Introduction:

In the NRL 94 GHz gyroklystron experiment, the input coupler is designed using a modern Maxwell's equation solver, HFSS. While this is a useful and accurate design tool, it has drawbacks in that it uses a very great amount of computer time. A typical run for the NRL input coupler typically takes several hours, and to calculate the coupling efficiency as a function of frequency typically takes many runs. This is only for one design; as parameters of the cavity, waveguide and coupling hole are varied, the amount of computer time can become very large indeed. Furthermore, there are other difficulties with HFSS, principally that it cannot easily calculate either the Ohmic Q of the cavities, or the beam loading of the main cavity. Usually these effects are added onto the solution after the fact. The purpose of this memo is do develop an approximate, but much simpler formulation for the coupling problem. This is to use the dipole approximation for the coupling apertures. The result is set of equations which can be numerically calculated very simply, so that many different runs can be done very quickly.

In our case, the input waveguide transmits a mode of unit amplitude into a coaxial coupling cavity. It excites the TE_{41} standing mode there. On the inner wall of the coaxial cavity are 4 slots to couple into the TE_{01} mode of the main cavity. However because of the symmetry, it is also possible that it will couple to the TE_{41} mode in the main cavity. This would be a competing mode. Thus there are 4 quantities to calculate, the reflection, the amplitude of the mode in the coaxial cavity, and the two possible modes in the main cavity. From this, we would also like to calculate the excitation of the beam.

The advantage of the dipole excitation theory is that it is capable of doing just this, and of calculating it very simply. The drawback is that it is not very accurate. The approximation is only valid if $ka<1$, where a the size of the coupling hole and k is the wave number of the mode. The dipole moment of the hole usually is proportional to a^3, so that once the approximation begins to break down, it loses accuracy very fast as a increases. However alternatively, there may be other coupling schemes with a larger number of smaller holes. In any case, because the theory is so relatively simple, and can be evaluated numerically very easily, it seems useful to develop this theory as an analytic guide to the coupling problem. At the very least it should be this, and possibly it will be more useful still.

It is also possible that the coaxial coupler can be regarded as a waveguide going around the main cavity, a waveguide which propagates modes in the positive and negative θ directions. Because these are traveling waves, the phases at each coupling hole now depend on the frequency of the input wave. Then the joint between the main input waveguide and the coaxial waveguide can be regarded as a waveguide T which can in turn be specified by a scattering matrix S. The key then is to design the T so as to optimize the coupling. As we will see, this means mainly designing the T to have the proper choice of direct reflection coefficient at the input waveguide. In terms of the

Manuscript approved September 11, 1996

scattering matrix, one can then solve the coupling problem. The most accurate solution would involve a reflection and transmission coefficient calculation every time the coaxial waveguide mode propagates across one of the coupling holes, (ie dipoles) to the main cavity. This is fairly complicated, although not nearly as complicated as solving the entire coupling problem with HSFF. In this memo we formulate a much simpler, but approximate approach to the coupling through a waveguide T, an approach which should take only seconds of computer time to evaluate. We model the excitation of the main cavity by the damping of the waveguide mode as it propagates in θ. By a simple energy conservation argument, we solve for this coupling coefficient, and thereby totally solve the coupling problem. This technique is only slightly more complicated than the dipole coupling problem

2. Coupling of Cavities and Waveguides

A. General Formulation

The input to the 94 GHz gyroklystron is coupled to input source through fundamental mode waveguide. This waveguide ends in a metal plate in which apertures are cut (the apertures may in fact be the entire waveguide area). On the other side of these apertures is a coaxial cavity, which itself encircles the main cavity. (This coaxial waveguide is approximately like a fundamental mode waveguide which makes a circle around the main cavity.) The joining of the waveguide and coaxial cavity is actually like a waveguide T. The cavities are tuned so that the TE_{41} standing mode in the coaxial cavity has the same frequency as the TE_{01} mode in the main cavity. Since the waveguide T couples to odd modes in E_r in the coaxial cavity, the coaxial mode has a node in E_r at the position of the waveguide coupling. Around the azimuth of the coaxial cavity are 4 equally spaced slots for coupling to the main cavity. As the main cavity only has a B_z at the wall, these slots are at maxima of B_z in the coaxial cavity. As these slots all have the same value of B_z in the coaxial cavity, it naturally couples to a TE_{01} mode in the main cavity. However, it might also couple to a standing TE_{41} mode in the main cavity also. A diagram of the coupling configuration, as well as the electric field configuration in a waveguide T is shown in Fig.(1).

We now derive the basic equations for the coupling. We first consider the cavities, then the waveguide. The cavity fields are governed by Maxwell's equations

$$\nabla \mathbf{x} \mathbf{B} = 4\pi \mathbf{J}/c - i(\omega/c)\mathbf{E} \qquad (1a)$$

$$\nabla \mathbf{x} \mathbf{E} = i(\omega/c)\mathbf{B} \qquad (1b)$$

where a time dependence exp-iωt has been assumed. We now assume that the field in the cavity can be expanded into a complete, orthogonal set of eigenfunctions of Eqs.(1 a and b) with J=0, and satisfying the boundary condition that E_{tan} and B_{nor} vanish on the cavity walls. These eigenfunctions are denoted by $\mathbf{E_n}$, and $\mathbf{B_n}$, where the eigenfunction is ω_n, and the normalization of the electric and magnetic fields is given by

$$\int d^3r\, \mathbf{E_n}^* \bullet \mathbf{E_{n'}} = \int d^3r\, \mathbf{B_n}^* \bullet \mathbf{B_{n'}} = V\delta_{nn'} \qquad (2)$$

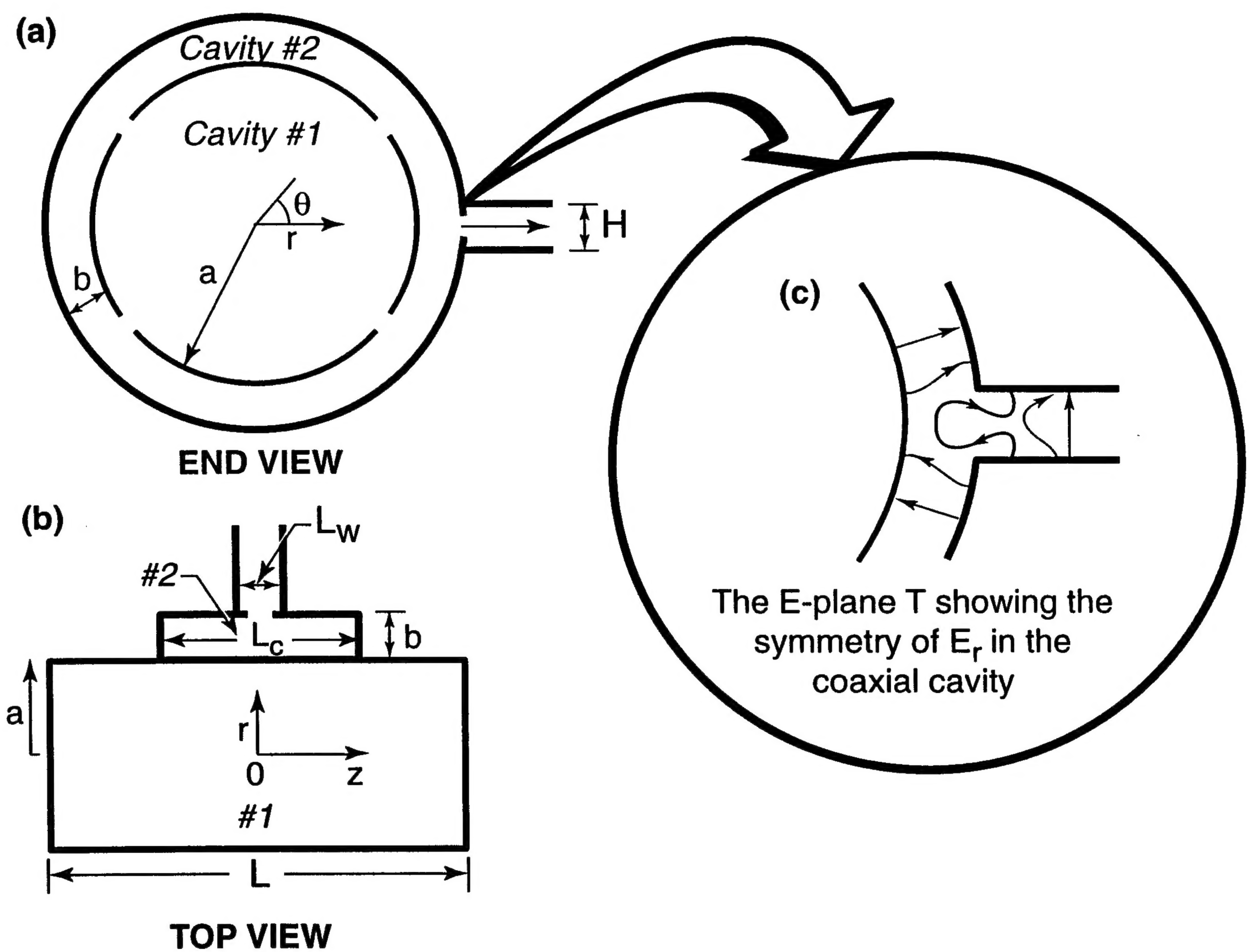
where V is the volume of the cavity. Thus, E_n and B_n are dimensionless, and we have used the fact that the electric and magnetic fields have the same stored energy in a cavity.

Take the dot product of Eq.(a) with E_n^* and Eq.(b) with B_n^*, subtract and get:

$$i\omega/c\{B_n^* \bullet B + E_n^* \bullet E\} + B_n^* \bullet \nabla x E - E_n^* \bullet \nabla x B = -(4\pi/c) J \bullet E \qquad (3)$$

Using standard vector identities, and also using Eqs.(1a and b) for the eigenfunctions, we find the result

$$i[(\omega-\omega_n)/c]\{E_n^* \bullet E + B_n^* \bullet B\} + \nabla \bullet [E x B_n^* - E_n^* x B] = -(4\pi J/c) \bullet E^*_n \qquad (4)$$



THE COUPLER

Figure 1

Now decompose the electric and magnetic field in the cavity into summations over the eigenfunctions

$$\mathbf{B} = \sum_n A_n \mathbf{B}_n \qquad \mathbf{E} = \sum_n A_n \mathbf{E}_n \tag{5}$$

Here, A_n is a scalar with dimension electric field and $\mathbf{B}_n$ and $\mathbf{E}_n$ are dimensionless vectors. Then integrate Eq.(4) over the volume of the cavity and find

$$2iV[(\omega-\omega_n)/c]A_n + \int_{walls} d\mathbf{S}\bullet(\mathbf{E}x\mathbf{B}_n{}^* + \mathbf{E}_n{}^*x\mathbf{B}) +$$

$$\int_{apertures} d\mathbf{S}\bullet(\mathbf{E}x\mathbf{B}_n{}^* + \mathbf{E}_n{}^*x\mathbf{B}) = -(4\pi/c)\int d^3r\mathbf{J}\bullet\mathbf{E}_n{}^* \tag{6}$$

The integral over the wall appears to vanish because both E and E_n vanish at a conducting wall. However, if the wall has some finite conductivity, E does not actually vanish at the wall, but this term in the wall integral represents the loss due to Ohmic dissipation in the wall. If only a single mode is excited in the cavity, this term represents the Ohmic losses for that mode. If more than one mode is excited, the dissipation does not necessarily separate out into a summation over the separate modes because there is in general no orthogonality relation between the surface integrals over the walls for the different eigenfunctions. That is there can be cross terms. However in practice, the eigenfunctions are often separable in one or more of the variables. For instance in our case, the main cavity is cylindrical, so that this cross term of the wall integral will vanish as long as the modes have different azimuthal or axial eigenvalues. There will only be cross terms between modes with different radial eigenvalues, but the same azimuthal and axial eigenvalue. For our case, we consider only the TE_{01}, and TE_{41} modes in the main cavity, so there is no cross term, and the Ohmic Q's of each mode can be considered separately. We also note that the Ohmic Q for modes due to wall resistivity is not pure real, in fact the real and imaginary parts are equal. This arises from the fact that at the surface of the conductor, the parallel electric field is given by $[\omega/8\pi\sigma]^{1/2}(1-i)\mathbf{n}x\mathbf{H}_p$ where σ is the conductivity of the wall and $\mathbf{H}_p$ is the parallel magnetic field at the wall. Thus if we denote Q_{oh} as a real quantity, the Ohmic Q, denoted Q_o is given by

$$Q_o = Q_{oh}(1+i) \tag{7}$$

where the sign of the imaginary part is such as to give a decrease in frequency.

Now consider the integral over the apertures. There are two separate apertures which we consider here, first the apertures which are the coupling holes, and secondly, other apertures. For instance theTE_{01} mode in the main cavity has an aperture representing the beam tunnel. These apertures give rise to loss and frequency shift for each mode, the diffractive Q of the mode. Again, we assume that the various symmetries allow us to consider the diffraction Q's for each mode without cross terms. We model the effect of these apertures and the wall losses with a total Q (not necessarily real) for all of them. The **J** on the term on the right hand side then represents the current from the electron beam. Hence, Eq.(6) reduces to

$$2iV[(\omega+i(\omega_n/2Q_n)-\omega_n)/c]A_n + \int_{apertures} d\mathbf{S}\bullet(\mathbf{E}x\mathbf{B}_n{}^*) = -(4\pi/c)\int d^3r\mathbf{J}\bullet\mathbf{E}_n{}^* \tag{8}$$

The surface integral in Eq.(8) now denotes an integral only over the coupling apertures and **dS** denotes an outward normal from the cavity. Also, in writing Eq.(8) above, we

have made use of the fact that E_n' vanishes in the aperture. Thus the integral over the aperture involves the exact electric field in the aperture dotted into the complex conjugate of the magnetic eigenfunction.

To continue, we work out a similar formulation for the waveguide modes. For the waveguide, the electric field is given as a summation over normal modes again, where the normal modes are normal modes of the transverse Laplacian. The dependence on z and t is given by $\exp i(\pm kz-\omega t)$. The plus sign is a mode going to the right, and the minus sign is a mode going to the left. They are independent solutions, but they have the same transverse eigenfunction for transverse electric field. In fact the transverse and longitudinal eigenfunctions for right and left propagating modes are related by

$$\mathbf{E}_{+\perp} = \mathbf{E}_{-\perp} \qquad \mathbf{B}_{+\perp} = -\mathbf{B}_{-\perp} \qquad\qquad \mathbf{E}_{+z} = -\mathbf{E}_{-z} \qquad \mathbf{B}_{+z} = -\mathbf{B}_{-z} \qquad (9)$$

Now say that the waveguide modes are also expanded in a complete orthogonal set. Let us denote the perpendicular field of the eigenfunction by $\mathbf{E}_{\perp wn}$ where w in the subscript denotes that it is a waveguide field rather than a cavity mode. Since the cavity is rectangular, the transverse mode in each direction is a standing mode, so we can assume without loss of generality that $\mathbf{E}_{\perp wn}$ is pure real. In this case, the normalization is chosen to be

$$\int d^2r\, \mathbf{E}_{\perp wn} \bullet \mathbf{E}_{\perp wn'} = A\delta_{nn'} \qquad (10)$$

The transverse magnetic field is given by $\mathbf{B}_{\perp wn} = \pm Z^{-1}\mathbf{i}_z \times \mathbf{E}_{\perp wn}$ where $Z=kc/\omega$ for TM mode, and $Z = \omega/kc$ for a TE mode. It is the impedance of the waveguide mode. We now assume that for the frequency considered, there is only one propagating mode in the waveguide. Now the transverse electric field in the waveguide is given by

$$\mathbf{E}_{\perp w} = [D^+ + D^-]\, \mathbf{E}_{\perp w1} + \Sigma_{n=2} D_n\, \mathbf{E}_{\perp wn} \qquad (11)$$

where the D's are z dependent coefficients and the E's are taken as the transverse field at the position of the screen or diaphragm in the waveguide. The D_1's account for the incident and reflected waves, and the other D's denote the evanescent waves. Far away, they are exponentially evanescent and negligible. As usual, the E's are dimensionless transverse vectors, and the D's are scalars with dimension electric field. Multiplying Eq.(11) by $\mathbf{E}_{\perp w1}$, integrating over a cross sectional area, and making use of the orthogonality relations, we find

$$D^+ + D^- = A^{-1} \int d^2r\, \mathbf{E}_{\perp w} \bullet \mathbf{E}_{\perp w1} \qquad (12)$$

Note that $\mathbf{E}_{\perp w}$ the exact transverse electric at the screen. Since it is zero on the conducting surfaces the integral in Eq.(12) is taken only over the apertures. $\mathbf{E}_{\perp w1}$, as always, is dimensionless. Since $\mathbf{E}_{\perp w1} = Z\mathbf{i}_z \times \mathbf{B}_{\perp w1}$, we find that

$$D^+ + D^- = Z A^{-1} \int_{\text{apertures}} (\mathbf{E}_{\perp w} \times \mathbf{B}_{\perp w1}) \bullet \mathbf{dS} \qquad (13)$$

where **dS** is an outward pointing normal. Thus the aperture coupling term for the waveguide modes is very much the same as that for the cavity modes.

B The Normalized Eigenfunctions and Eigenvalues

Here we specify the normalized fields for the various cavities and waveguides. The main cavity will be specified with a subscript 1, and the coaxial cavity with the subscript 2. In the main cavity we consider only the TE_{01} and TE_{41} modes, so these are denoted with subscripts 01 and 04. In the coaxial cavity, we consider only the TE_{41} mode, so this is denoted with simply the subscript 2. We also consider only the fundamental TE_{01} waveguide mode, and for this we use the single subscript w. The inner cavity has a radius a and length L, extending from -L/2 to +L/2. Considering only the lowest order axial mode, we find for a mode with azimuthal mode number p,

$$\mathbf{E}_{1np} = K(\cos\pi z/L)\, i_z x \nabla x \Psi, \text{ and } \Psi = J_p(k_{np}r)\cos p\theta \tag{14a}$$

where we have assumed that the theta dependence is a standing mode and $J_p(k_{np}a)=0$. Here, $k_{np}^2 = (\omega/c)^2 - (\pi/L)^2$. The $\cos\theta$ choice (rather than $\sin\theta$) is the proper orientation for the coupler we are using. Also, for the modes we will be considering, p is either 0 or 4. The coefficient K is determined so that the eigenfunction is properly normalized. Using integral relations for the Bessel functions, we find

$$K = 2K_p a/M, \text{ where } K_p = 2^{1/2} \text{ for } p\geq 1, \text{ and } K_p = 1 \text{ for } p=0, \text{ and}$$

$$M = (x_{np}^2 - p^2)^{1/2} J_p(x_{np}) \tag{14b}$$

where x_{np} is the n^{th} zero of J_p' (that is $k_{np}=x_{np}/a$). Since it is a TE mode, this is the only component of the electric field. The magnetic field is given by

$$\mathbf{B}_{1\perp np} = -K(\pi ic/\omega L)(\sin\pi z/L)\, \nabla\{ J_p(k_{np}r)\cos p\theta\} \tag{14c}$$

and

$$B_{1znp} = K(ic/\omega)[(\omega/c)^2 - (\pi/L)^2](\sin\pi z/L)\, J_p(k_{np}r)\cos p\theta \tag{14d}$$

Now we consider the coaxial waveguide. It is a TE mode in the cavity, where B_z is a linear combination of Bessel and Neuman functions. For the lowest order axial mode, the dispersion relation is

$$J_p'(ka)N_p'(k[a+b]) - J_p'(k[a+b])N_p'(ka) = 0 \tag{15}$$

where as before, $k^2 = (\omega/c)^2 - (\pi/L_c)^2$. The eigenfunction is a standing wave in θ with a node in E_r at θ=0. We approximate it as a fundamental wave in a (bent) waveguide, so

$$E_{2r} = (4/K_p)^{1/2}\sin p\theta\cos(\pi z/L_c) \quad (16a)$$

$$B_{2z} = -(4/K_p)^{1/2}(ipc/[a+b/2])\cos p\theta\cos\pi z/L_c \quad (16b)$$

$$B_{2\theta} = -(4/K_p)^{1/2}(\pi ic/\omega L_c)\sin p\theta\sin(\pi z/L_c) \quad (16c)$$

where as before, $K_p = 2^{1/2}$ for $p\geq 1$, and $K_p = 1$ for $p=0$. If one assumes the eigenfunction is as a fundamental mode in a waveguide, an approximate eigenvalue is $\omega^2 = (\pi c/L_c)^2 + (pc/[a+b/2])^2$. For the coaxial mode, we only consider p=4.

Finally we consider the fundamental mode waveguide. As the waveguide is oriented, r is what would usually be the z component of the guide, and θ and z are the two perpendicular components. For the waveguide oriented with respect to the coaxial cavity so that it makes an E plane T, we have

$$E_{w\theta} = 2^{1/2}\cos(\pi z/L_w) \qquad B_{wz} = -(\pm)\,(kc/\omega)E_{w\theta} \qquad B_{wr} -i(\pi c/\omega L_w)\,E_{w\theta} \quad (17)$$

Here, the $\pm$ denotes the incident or reflected wave into the cavity. The overall minus in the expression for B_{wz} arises because in the configuration shown in Fig. (1), the incident wave travels to the left. The dispersion relation for the waveguide is $\omega^2 = (kc)^2 + (\pi/L_w)^2$.

This then specifies the eigenfunctions and eigenvalues for the main cavity, the coaxial cavity and the waveguides.

C. The calculation of $-(4\pi/c)\int d^3r\mathbf{J}\bullet\mathbf{E}_n^*$

To calculate the above quantity, we must calculate the current generated on the electron beam, by the fields in the waveguide. As we consider only two modes in the main cavity, the TE_{01} and the TE_{41} which have different θ symmetries, there is no current generated by one mode which affects the other, at least in linear theory. Also, since the beam is placed to maximize the interaction with the TE_{01} mode, and thereby has a weaker interaction with the TE_{41} mode, we consider only the current generated by the former. Naturally the beam current has no effect on the coaxial cavity or waveguide mode. To

calculate the current, we use the linearized Vlasov equation. To simplify this, we assume the beam is only weakly relativistic, so that the current is generated only by the electric field of the cavity mode. In the transverse plane, we use as independent variables the guiding center positions and the perpendicular coordinates in momentum space. That is the momentum variables are p_z, $p_\perp$, and ϕ where ϕ is an azimuthal angle in momentum space. That is

$$p_r = p_\perp \cos(\phi+[\Omega mz/p_z]-\theta) \quad \text{and} \quad p_\theta = p_\perp \sin(\phi+[\Omega mz/p_z]-\theta) \qquad (18)$$

where θ is the azimuthal angle in real space and Ω is the nonrelativistic cyclotron frequency of the electron. In these coordinates, the linearized Vlasov equation is

$$[-i\omega+(p_z/\gamma m)\partial/\partial z]f = e\mathbf{E}\bullet\partial f_0/\partial\mathbf{p} = 2eE_\theta p_\theta \partial f_0/\partial p_\perp^2 \equiv G(z) \qquad (19)$$

where f_0 is the unperturbed distribution function. For the TE_{01} mode in the main cavity, the right hand is $2eE_\theta p_\theta \partial f_0/\partial p_\perp^2$, where we have made use of the assumption that the unperturbed distribution function does not depend on ϕ. The solution to Eq.(19) is

$$f = \exp[i\gamma mz/p_z]\int_0^z dz' \exp[-i\gamma mz/p_z]G(z') \qquad (20)$$

and the current in the θ direction is

$$J_\theta = -\int d^3p(ep_\theta/\gamma m)f. \qquad (21)$$

and what we want is $-(4\pi/c)\int d^3r\mathbf{J}\bullet\mathbf{E_n}^*$. This is

$$-(4\pi/c)\int d^3r\mathbf{J}\bullet\mathbf{E_n}^* = (4\pi e^2c/\gamma m)\int d^3r d^3p\int_0^z dz p_\perp^2 \partial f_0/\partial p_\perp^2 A_{10}E_{10}^*(r,z)\exp i\alpha(z-z')E(r,z') \quad (22)$$

where $\alpha=(m/p_z)[\gamma\omega-\Omega]$, A_{10} is the amplitude of the TE_{10} mode in the main cavity (having dimension electric field), and E_{10} is the normalized, dimensionless eigenfunction of that mode. Also, we assume that in E_{10}, the particle position and guiding center position are the same. Furthermore, in writing Eq.(22), we have assumed that only the slowly varying terms in the oscillating exponential integral contribute. These are all standard approximations in the calculation of oscillating currents in gyrotrons at the cyclotron frequency. To proceed, we assume that the unperturbed distribution function is given by

$$f_0 = (I/2\pi r_b ev_z)[\delta(r-r_b)/2\pi]\{\delta(p_z-p_{z0})\delta(p_\perp^2 - p_{\perp 0}^2)/\pi\} \qquad (23)$$

where I is the beam current and r_b is the beam radius. Furthermore, to make the result somewhat simpler analytically, we approximate the z dependence of the eigenfunction

with $\pi^{-1/2}\exp-(z/L)^2$, where the lower limit of the z integral is not taken as minus infinity. The z and z' integrals can be done almost analytically. The result is

$$\pi^{-1}\int_{-\infty}^{\infty}dz\int_{-\infty}^{z}dz'\ \exp-[(z^2+z'^2)/L^2\ +i\alpha(z-z')]\ =0.5L^2\{\exp-(\alpha L)^2/2\ \ -iG(\alpha L)\} \tag{24}$$

where

$$G(y) = [1/2\pi^{1/2}]\exp-(y^2/2)\int_{-\infty}^{\infty}du(\exp-u^2)\ (\mathrm{erf}u)(\sin 2yu) \tag{25}$$

where G is a function that must be evaluated numerically. Putting this together, we find that the right hand side of Eq.(22) can be included in the left hand side as an effective Q_B, the reciprocal of which can be added to all other relevant reciprocal Q', for instance the Ohmic Q. We find

$$1/2Q_B = [IeL/\omega_{10}m\gamma v_{zc}^{\ 2}\pi a^2\}\,|E_\theta(r_b)|^2\,\partial/\partial p_\perp^{\ 2}\{\,p_\perp^{\ 2}[\exp-(\alpha L)^2/2 - iG(\alpha L)]\} \tag{26}$$

in taking the derivatives with respect to $p_\perp^2$, notice that α depends on $p_\perp^2$ so that

$$\partial/\partial p_\perp^{\ 2} = [m\omega_{10}L/p_z\gamma(mc)^2\,]\partial/\partial\alpha L \tag{27}$$

Also, all p's are evaluated at the unperturbed beam momentum. Notice that the real part of Q_B has a negative contribution for positive α (frequency above the Doppler shifted cyclotron frequency) as is expected for a gyrotron. Also it has an imaginary part corresponding to the beam generated frequency shift of the TE_{01} mode in the main cavity. By incorporating the Q_B in the formulation, on can account for the beam loading in a straightforward way within the total formulation of the coupling problem.

D. Dipole Approximations to the Aperture Couplings

We now turn to an evaluation of the $\int_{\mathrm{apertures}}\mathbf{dS}\bullet(\mathbf{E}\mathrm{x}\mathbf{B}_n{}^*)$ term in the equation for A_n or D. The dipole approximation consists of two parts, first assuming that the coupling apertures are very small compared to k^{-1}, and secondly, assuming that the resulting integrals can be approximated as those from the electrostatic or magnetostatic approximation. The quantity $B_n{}^*$ are known from the knowledge of the normalized eigenfunctions. Assuming that the coupling aperature is small, this can be approximated as

$$\mathbf{B}_n{}^* = \mathbf{B}_n{}^*(0) + \qquad \Sigma_\beta\, x_\beta\, \partial/\partial x_\beta\, \mathbf{B}_n{}^*(0) \tag{28}$$

The first term simply gives

$$\int_{apertures} d\mathbf{S}\bullet(\mathbf{E}\mathrm{x}\mathbf{B}_n^*) = \int_{apertures} \mathbf{B}_n^*(0) \bullet [\mathbf{n}\mathrm{x}\mathbf{E}]\ da \qquad (29)$$

where n is the unit normal to the aperture, and da is now the scalar unit area we are integrating over. Here, the term in the brackets is the exact tangential component of the electric field within the aperture. It clearly plays the role of an effective magnetic moment of the aperture. Note also that in Eq.(29), the unit vector **n** is defined as the outward pointing normal. Thus, in considering the dipole approximation to whatever is on the other side of this cavity (another coupling cavity or waveguide), its unit vector also points outward (into the first cavity) , so that its effective magnetic moment changes sign.

Now let us consider the next term in the approximation. It is

$$\int_{apertures} d\mathbf{S}\bullet(\mathbf{E}\mathrm{x}\textstyle\sum_\beta x_\beta\, \partial/\partial x_\beta\, \mathbf{B}_n(0)^*)$$

As in Jackson, it is convenient to separate this into an anti-symmetric and a symmetric part. Using the complete anti-symmetric tensor to describe the cross product, assuming summation over repeated indices, and for convenience dropping the (0) and mode index n, we find the result is

$$\int_{apertures} da\ n_i\varepsilon_{ijk}E_j x_\beta\{[\partial B_k/\partial x_\beta - \partial B_\beta/\partial x_k] + [\partial B_k/\partial x_\beta + \partial B_\beta/\partial x_k]\} \qquad (30)$$

As Jackson shows, the symmetric part contributes to a higher multipole moment. Using Maxwell's homogeneous equation, the antisymmetric part reduces to

$$i\omega_n/2c\int_{apertures} da\ n_i\varepsilon_{ijk}E_j x_\beta E_{n\alpha}^*(0)\varepsilon_{\alpha\beta k} = -i\omega_n/2c\int_{apertures} da\ \mathbf{E}_n^*(0) \bullet\{\mathbf{n}[\mathbf{E}\bullet\mathbf{x}]\} \qquad (31)$$

Clearly the quantity dotted into E_n^*, which is oriented normal to the aperture, plays the role of an effective electric dipole moment. By the same logic as with the magnetic moment, it changes sign when going from one side of the aperture to the other. Thus the aperture coupling terms can be approximated as

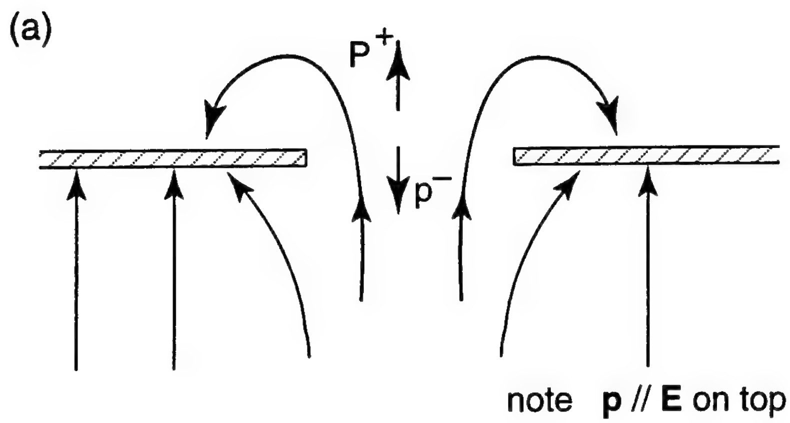
$$-2\pi i\omega_n/c[\mathbf{p}_{ef} \bullet\mathbf{E}_n^*(0) - \mathbf{m}_{ef}\bullet\mathbf{B}_n^*(0)] \qquad (32)$$

where

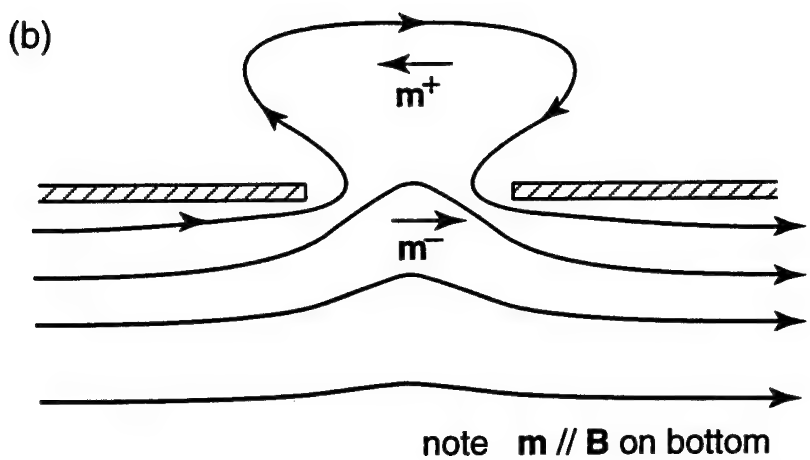
$$\mathbf{p}_{ef} = n/4\pi\int_{apertures} da\ \mathbf{E}\bullet\mathbf{x} \qquad \text{and} \qquad \mathbf{m}_{ef} = c/2\pi i\omega\int_{apertures} da\ \mathbf{n}\ \mathrm{x}\ \mathbf{E} \qquad (33)$$

In the expressions for p_{ef} and m_{ef}, it is only the component of the electric field tangential to the aperture that contrubutes. In each case, it is the exact electric field.

The small aperture coupling problem then reduces to calculating the electric and magnetic dipole moments. As Jackson points out, for kx<<1, the solution to Maxwell's equations near the hole reduces to the solution of the electrostatic or magnetostatic problem. Imagine a small hole in an infinite conducting sheet with a static electric field perpendicular to the sheet on one side, and a different value of the field on the other side.



The effective electric dipole movment of an aperture with a discontinuity of $\mathbf{E}_{nor}$



Effective magnetic dipole movement of an aperture with a discontinuity of $\mathbf{B}_{tan}$

Figure 2

For simplicity, we consider the case of the field vanishing above the screen, as shown in Fig (2A). The field has a configuration of a dipole moment in the middle of the hole, but with one variation. For a true dipole, the tangential electric fields above and below the sheet would be anti-parallel, whereas the in Fig (2A), the outward radial fields above and below the aperture are both outward, that is the sign of the dipole changes above and below the hole.

The case of a magnetic dipole, for a magnetic field tangential to a conducting sheet with an aperture is quite analogous. Here there are different magnetic fields above and below the sheet. The configuration is shown in Fig (2B). It is quite analogous to the electric case, except the overall sign of the magnetization is reversed. The electrostatic or magnetostatic problem of the infinite conducting sheet with apertures, which Jackson solves for a circular hole, then can be used to define the effective dipole moments. The effective electric moment is given by an electric polarizability P time the difference in normal electric field at the surface. Here P is a scalar. The magnetic polarizability is given by a magnetic polarizability **M** times the difference in tangential magnetic field at the surface. Here **M** two dimensional tensor, because the tangential field is a two dimensional vector. Thus

$$\mathbf{p}_{ef} = P\Delta\mathbf{E}_{nor} \quad \text{and} \quad \mathbf{m}_{ef} = \mathbf{M} \bullet \Delta\mathbf{B} \tag{34}$$

For a circular hole of radius R, Jackson gives the result

$$P = -R^3/3\pi \qquad\qquad M = 2R^3/3\pi \tag{35}$$

where for the circular hole, M is a scalar. For an elliptical aperture shown in Fig.(3), Gao shows that the polarizability is given by

$$P = -\pi l_1^3(1-e_0^2)/3E(e_0) \tag{36a}$$

$$M_{11} = \pi l_1^3 e_0^2/3[K(e_0)- E(e_0)] \tag{36b}$$

$$M_{22} = \pi l_1^3 e_0^2(1-e_0^2)/3[E(e_0)- (1-e_0^2)\, K(e_0)] \tag{36c}$$

$$M_{12} = M_{21} = 0 \tag{36d}$$

where e_0 is the eccentricity of the ellipse, $e_0 = (l_1-l_2)/l_1$, and the K's and E's are the complete elliptic integrals of the first and second kind. The expressions for elliptical apertures may be useful approximations to the polarizabilities of the rectangular apertures used in the NRL gyroklystron experiment. Generally, calculating these P's and M's are quite difficult. They involve solving Laplace's or Maxwell's equations with mixed boundary conditions. Other approximations have been given for corrections due to the finite thickness of the sheet for a circular aperture, and results have also been tabulated for electromagnetic effects in a circular hole in a sheet of non-zero thickness[4].

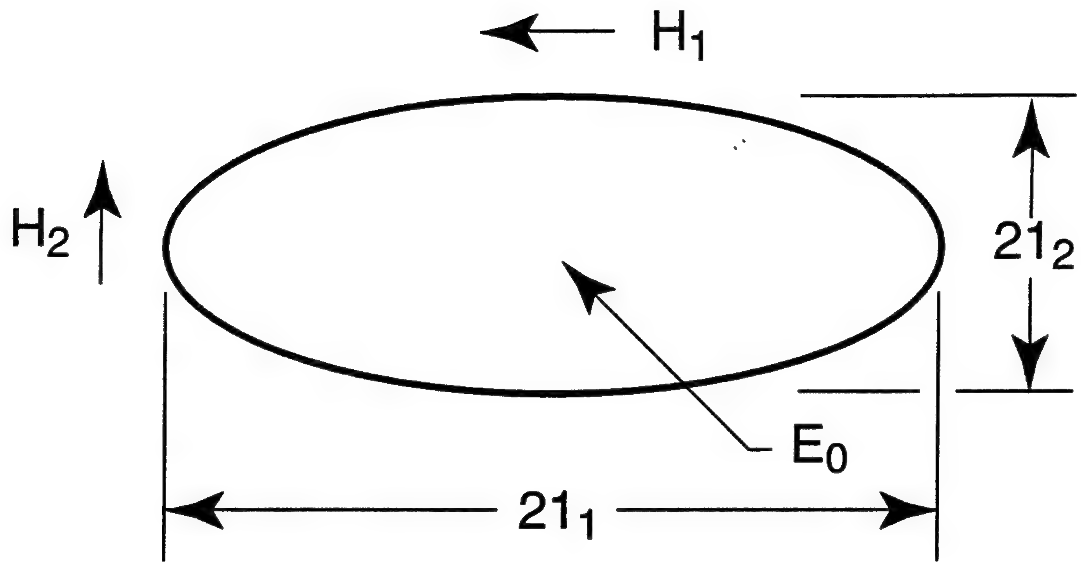


Figure 3

3. Simple Waveguide Coupling to a Cavity.

We consider here the simplest case of a fundamental mode waveguide coupling to a cavity which can have only a single mode in it. The waveguide runs perpendicular to the cavity wall and ends there in a wall with an aperture in it. The coupling is through this aperture, which is assumed small enough that the dipole approximations are valid. The value of a field quantity at the coupling hole is then given by the mode amplitude (A for the cavity and D for the waveguide) times the appropriate component of the normalized eigenfunction (that is the normal electric field or the tangential magnetic field) at the center of the coupling hole. Since the rectangular waveguide propagates only fundamental mode, there is only a single component of B tangential to the screen between the cavity and waveguide, and the component of E normal is zero for a TE mode. If the aperture is elliptical, with its axis oriented along the magnetic field, the magnetization then is a scalar as far as the waveguide is concerned. We will assume that only that component of magnetic field is excited in the cavity at the coupling hole, but will discuss later the effect of other magnetic components and the normal electric field in the cavity.

For the normalized field components, we will use the notation of a subscript w for waveguide fields, and no subscript for cavity fields. Also, where we us a B or E, it is assumed to be at the center of the coupling hole. Then the equation for the cavity field is

$$(\omega + i\omega_0/2Q - \omega_0)A = 2\pi\omega/V\{M[(D^+ - D^-)B_w - BA]\}B^* \tag{37}$$

where ω_0 is the frequency of the cavity mode. On the right hand side of Eq.(37) above, the D's are subtracted because the magnetic coupling involves the tangential magnetic fields i the waveguide. Since the D's represent electric fields, the magnetic field of the forward and backward propagating modes have opposite signs. The equation for the waveguide modes is

$$D^+ + D^- = -2i\omega Z/\alpha c\{ M[(D^+ - D^-)B_w - BA]\}B_w \tag{38}$$

where, recall Z is the impedance of the waveguide mode, and to avoid confusion with the cavity mode amplitude, α is now the cross sectional area of the waveguide. Thus we have two equations for the coupling of the cavity and waveguide mode.

When looking at the cavity mode, one effect is the self interaction term on the right hand side. Since this term is pure real, it corresponds to a frequency shift resulting from the coupling hole. Clearly it is pure real, and in addition, the signs are such that it corresponds to a decrease in frequency, since M>0. If the cavity mode had the other component of the magnetic field, or else a normal component of the electric field, this result would not have changed. The additional terms in the cavity mode equation, Eq.(37) would still correspond to a frequency decrease.

To start, we consider the case where there is no incident electric field. In this case, there are two homogeneous equations for the two unknowns A and D^-. For the case of small M, the dominant effects are the frequency shift just discussed, which is proportional to M, and a damping, proportional to M^2, which is

$$\nu = 4\omega^2 Z M^2 B_w{}^2 |B|^2/\alpha V[1-2i\omega M B_w{}^2/\alpha] \tag{39}$$

Since ν has an imaginary part as well, there are other corrections to the frequency shift proportional to higher powers of M. To lowest order in M, the coupling to the waveguide may be modeled by coupling Q, which is

$$1/2Q_c = 4\omega Z B_w{}^2 M^2 |B|^2/\alpha V \tag{40}$$

Finally, it is not difficult to show that if there is an incident wave, $D^+ \neq 0$, and the cavity is characterized by $Q=\infty$, the reflection coefficient D^-/D^+ has magnitude unity. If $Q \neq \infty$, then one can also show very simply that the reflection coefficient is zero if $Q=Q_c$. These are both standard results. Hence when coupling to a cavity, in the power input into the cavity maximizes for a coupling hole radius such that $Q=Q_c$, that is for critical coupling. If the coupling hole is too small, the cavity is under-coupled and power will be reflected. Also if the coupling hole is too big, the cavity is over-coupled, and power will be reflected. In practice, usually the coupling hole is drilled small, and is slowly increased in size until critical coupling is achieved. Often the dipole approximation is not well satisfied at critical coupling, and the empirical approach works better.

4. Coupling of Cavities to Each Other.

Now let us consider the other case of cavities coupled to each other but not to the outside world. Say that a mode in cavity 0 is connected to one of n modes in cavity 1, and vise versa. Then we have for the modes in cavity 0,

$$V_0(\omega + i\omega_{0n}/2Q_{0n} - \omega_{0n})A_{0n} = 2\pi\{-B_{0n}{}^*M(B_{0n}A_{0n} - \Sigma_m B_{1m}A_{1m}) + E_{0n}{}^*P(E_{0n}A_{0n} - \Sigma_m E_{1m}A_{1m})\} \tag{41a}$$

and correspondingly for the modes in cavity 1,

$$V_1(\omega + i\omega_{1n}/2Q_{1n} - \omega_{1n})A_{1n} = 2\pi\{-B_{1n}{}^*M(B_{1n}A_{1n} - \Sigma_m B_{0m}A_{0m}) + E_{1n}{}^*P(E_{1n}A_{1n} - \Sigma_m E_{0m}A_{0m})\} \tag{41b}$$

The self interaction terms on the right hand side of Eqs. (41a and b) all give rise to a decrease in the mode frequency (recall that M is positive and P is negative).

Now consider the cross terms. If we redefine the independent variables as $V^{1/2}A$, we can divide Eq.(41a) by $V_0{}^{1/2}$ and Eq(41b) by $V_1{}^{1/2}$, then in both Eqs.(41a and b), the quantity multiplying the dependent variable is simply for instance $\omega + i\omega_{1n}/2Q_{1n} - \omega_{1n}$. The off diagonal terms on the right sides of Eqs.(41a and b) are then divided by $[V_0V_1]^{1/2}$, so that the Hermitian property of the operator on the right hand side is preserved. Thus Eqs.(41a and b) is a simple eigenvalue equation with a Hermite operator on the right hand sides. Since a Hermitian operator has real eigenvalues, the coupling of the cavities to one another give rise to frequency shifts among the normal modes, but do not give rise to growth or damping. This is of course what is expected.

5 Waveguide and Multi-cavity Coupling.

For the case of the waveguide (denoted w) coupled to the coaxial cavity (denoted 2), which can itself couple to the main cavity in either the TE_{01} mode (denoted 10), or TE_{41} mode (denoted 14), there are 4 equations which describe the coupling. These are

Waveguide to cavity 2:

$$D^+ + D^- = -2i\omega Z/\alpha c\{ M[(D^+ - D^-)B_{wz} - B_{2z}A_2]\}B_{wz} \tag{42a}$$

Here we note that the waveguide mode only has a B_z which can couple to the cavity. The waveguide eigenfunction is given in Eq.(17). Also we assumed that the M tensor is either a scalar or else has principle axes in the z and θ directions. The next is

Cavity 2 to cavity 1 and the waveguide:

$$\begin{aligned} V_2(\omega + i\omega_2/2Q_2 - \omega_2)A_2 = 2\pi\{&M_{zz}[(D^+ - D^-)B_{wz} - B_{2z}A_2]\}B_{2z}^* + P|E_{2r}|^2 A_2 \\ &+ M_{\theta\theta}|B_{2r}|^2 A_2 + M'_{\theta\theta}|B'_{2r}|^2 A_2 + P'|E'_{2r}|^2 A_2 + \\ &M'_{zz}B'^*_{2z}[B'_{2z}A_2 - B'_{01z}A_{10} - B'_{14z}A_{14}] + M'_{\theta\theta}B'^*_{2\theta}[B'_{2\theta}A_2 - B'_{14\theta}A_{14}] \\ &-P'E'^*_{2r}[E'_{2r}A_2 - E_{14r}'A_{14}]\} \end{aligned} \tag{42b}$$

Cavity 1, TE_{01} mode:

$$V_1(\omega + i\omega_{10}/2Q_{10} - \omega_{10})A_{10} = 2\pi\{-M'_{zz}[B'_{10z}A_{10} + B'_{14z}A_{14} - B'_{2z}A_2]B'^*_{10z}\} \tag{42c}$$

Cavity 1, TE_{14} Mode:

$$\begin{aligned} V_1(\omega + i\omega_{14}/2Q_{14} - \omega_{14})A_{14} = 2\pi\{&- M'_{zz}[B'_{10z}A_{10} + B'_{14z}A_{14} - B'_{2z}A_2]B'^*_{14z} \\ &+ M'_{\theta\theta}B'^*_{14\theta}[B'_{2\theta}A_2 - B'_{14\theta}A_{14}] - P'E'^*_{14r}[E'_{2r}A_2 - E_{14r}'A_{14}]\} \end{aligned} \tag{42d}$$

Here, an unprimed value of P, M or eigenfunction means that these quantities are evaluated at the coupling hole between the waveguide and the coaxial cavity, and a primed value means these quantities are evaluated at the coupling hole between the

coaxial and main cavity. With the nature of the coupling chosen, these quantities are the same at all four coupling holes between the coaxial and main cavity.

Thus we have 4 simultaneous equations for the 4 modes. Let us specify D^+, then they can be solved for the four amplitudes as a function of frequency. Also they are simple enough to solve numerically that many parameters can be easily varied. While they will not give exact solutions due to the fact that the dipole approximation may not be valid for coupling holes of the size that may be used in the experiment, they may well give important in into the nature of the solutions and their scaling with many parameters.

6. The Coupling at a Waveguide T.

The coupling of the waveguide to the coaxial mode, as shown in Fig. 1 is actually like a waveguide T. If the scattering matrix S of the T is specified, one can solve for the outgoing waves produced by a any incident wave. Waveguide T's are discussed in standard texts[5,6] and they have certain symmetry and orthogonality properties. If the field amplitude in each arm of the T is normalized so that the power flux in the arm is proportional to the field amplitude squared (with the same coefficient of proportionality in each arm of the T), then the S is a symmetric matrix. Also, of the T is lossless and passive, energy conservation gives two additional relations. The first is that the sum of the squares of the magnitudes of each column is unity. The second is that the dot product of any column with the complex conjugate of any other column is zero.

In our configuration of the waveguide T, we will let 3 be the port of the input wave, 1 be the port of the wave going in the positive θ direction, and 2 be the port of the wave going in the negative θ direction. We assume only that the T, as shown in Fig 1 is symmetric with respect to up-down reflection. This means that $S_{13} = -S_{23}$ and $S_{11} = S_{22}$. Thus the scattering matrix, valid for any symmetric T has the form

$$\begin{matrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & -S_{13} \\ S_{13} & -S_{13} & S_{33} \end{matrix}$$

Let us say that the direct reflection in port 3 has magnitude ц, or in other words

$$S_{33} = \text{ц} \exp i\text{ж} \qquad (43)$$

where $0 < \text{ц} < 1$. Then $S_{13} = K \exp i\beta$ where $[(1-\text{ц}^2)/2]^{1/2} = K$. The unit magnitude of the second column gives the result

$$|S_{12}|^2 + |S_{11}|^2 + K^2 = 1 \qquad (44)$$

and the orthogonality relation between the rows gives

$$S_{12} = S_{11} + \text{ц} \exp i(-\text{ж}+2\beta) \qquad (45)$$

The scattering matrix relates the amplitude of the incident field in each leg to the outgoing field in all legs. Let us specify the incident and outgoing fields in the input waveguide as D^+ and D^- as in previous sections. The incident and outgoing fields in the circular waveguide are specified as F_1^+, F_1^-, F_2^+ and F_2^-. They are normalized so that the power fluxes are equal to a single constant time F or D squared. We will specify the normalizations in terms of actual fields later.

The resonator is a ring so the incident field into leg number one is simply related to the outgoing field in leg two. In fact, we specify that

$$F_1^+ = (F_2^-)g\exp i\lambda \qquad (46)$$

and the same for F_1^- and F_2^+. The λ describes the propagation of the wave around the ring resonator, and the g describes any damping along the way, for instance from Ohmic dissipation or excitation of the waves in the main cavity. Then the first two equations of the scattering matrix can be solved for the amplitudes of the outgoing fields in legs one and two in terms of the incident field in leg 3. The result is

$$F_1^- = -F_2^- = KD_+\exp i\beta/[1-g\ \text{ц}\ \exp -i(\gamma-\lambda)] \qquad (47)$$

where γ = 2β-ж. If both g and ц are close to unity, the denominator can have nearly resonant behavior if γ-λ=2πn where n is an integer. This means that a wave can be trapped in the resonator. However as ц approaches one, K approaches zero as (1- ц^2)$^{1/2}$, so the mode amplitude approaches infinity as ц approaches 1 no faster than (1- ц)$^{-1/2}$. However as ц approaches one, the coupling hole gets smaller, and the dipole approximation, which explicitly displays the resonant behavior gets more and more accurate. Once the outgoing waves in each leg are known, the incoming waves are also known via Eq.(46).

From the three incoming waves, we can find the reflection in leg 3, which is, R= D_-/D_+. It is

$$R = \exp i(\text{ж} -\gamma+\lambda)\ \{\ [\text{ц}\exp i(\gamma-\lambda)-g]/[1-g\ \text{ц}\exp -i(\gamma-\lambda)]\} \qquad (48)$$

Clearly if g=1, or no dissipation on the wave as it goes around the waveguide, the reflection coefficient R has magnitude unity. On the other hand if g is less than unity, the waveguide T can also be designed so that the total reflection coefficient R vanishes. Since g and ц are both real, positive and less than unity, the numerator can vanish (ie at critical coupling) but the denominator cannot. The condition for critical coupling is

$$g=\text{ц} \quad \text{and } \gamma-\lambda = 2\pi n \qquad (49)$$

7. Coupling to the Main Cavity Through a Waveguide T

We would now like to develop a simple theory for the coupling to the main cavity. As the wave propagates from the T it passes one of he coupling holes for which the dipole theory is assumed to be valid. From the dipole moments of the hole and the field amplitudes in the circular waveguide and in the main cavity, we can calculate the reflection and transmission. This is turns out to be a matrix a matrix multiplication, but one that involves the main cavity field as well as the transmitted and reflected field. Going to the next coupling hole we get another similar matrix multiplication, and so on until we get back around to the T. Here various input and output waves are then related by the scattering matrix, and one can solve for all field amplitudes by inverting this series of matrix multiplication. However here we formulate a much simpler, but approximate scheme, which gives an approximate solution using only conservation of energy.

Before we begin however, we must properly normalize the waveguide modes and relate them to the fields in the waveguide. The Power flow in the input waveguide (leg 3 of the T) is given by the integral of the Poynting vector. It is

$$P = cHL_w \, |D|^2/4\pi Z \qquad (50)$$

where all notation is defined in Sec 2B. Now consider the coaxial waveguide with the modes traveling in the plus or minus θ direction. The fields are defined as in Eq.(17) except that

B_r becomes B_θ ; E_θ becomes E_r; B_z remains B_z

We want to define the mode amplitude so that the power flux is $cHL_w |F|^2/4\pi Z$. However the coaxial waveguide might have different impedance and area, so that F does not bear the same relation to the fields in the waveguide that D does in Eq.(17). If F' is related to the fields as in Eq.(17) for the main input waveguide, then the power flux is $cHL_c |F'|^2/4\pi Z_c$, where Z_c is the impedance of the coaxial waveguide. Thus F and F' are related to each other by

$$F' = F[Z_c HL_w/ZbL_c]^{1/2} \qquad (51)$$

Now let us parameterize the coaxial waveguide mode with a single parameter. Let us say that the θ dependence is given by $\exp[i\omega\rho/c - Б]\theta$ for the mode traveling in the positive θ direction, and $\exp-[i\omega\rho/c - Б]\theta$ for the mode traveling in the negative θ direction. Here $\rho = a + b/2$. This approximates the mode in the circular waveguide with the approximation for a linear waveguide with $\rho\theta$ playing the role of distance z along the axis of the waveguide. Also, ω/c is the free space wave number of the wave. Note that as ω varies through the coupling range, the phase of the driving wave at each dipole is now approximately accounted for. The quantity Б represents the damping of the wave as it propagates in θ; the damping is caused by excitation of the mode in the main cavity as well as damping due to Ohmic dissipation in the waveguide walls (which we do not

consider here). We expect this model to be reasonably valid if $2\pi\,\text{Б} \ll 1$. If this is violated, at the very least, there would be corresponding a correction to the free space wave number which would also modify the phase of the wave at each coupling hole.

In terms of this quantity Б, one can calculate the reflection coefficient R of the input wave. That is R is given by Eq.(48), where $g = \exp{-2\pi\text{Б}}$, and $\lambda = 2\pi\omega\rho/c$. From the given value of Б and the waveguide T relations, we can find the propagating mode in the coaxial waveguide as a function of θ. The amplitudes of the outgoing modes at the waveguide T is F_1^- and F_2^-, where these are given in Eq.(47). Then, as a function of θ, the mode amplitude is given by

$$F(\theta) = F_1^-\{ \exp[i\omega\rho/c - \text{Б}]\theta - \exp([-i\omega\rho/c + \text{Б}\,]\theta - 2\pi\text{Б})\},\quad 0< \theta < 2\pi \qquad (52)$$

To get the field components, we first find F' in terms of F, and then from the F', we find the various field components with the analogous relation to Eq.(17). Once we have these field components in the coaxial waveguide (in terms of the parameter Б), we can rewrite Eqs. (42c and d), but with the waveguide fields. This is a pair of linear equations for A_{10} and A_{14} and they can be easily solved in terms of D^+ and Б.

Once the field amplitudes in the main cavity are known, the power dissipated ineach mode is given by $\omega V_1 A^2/Q$. Unless ц is very near 1 and the mode is very resonant (in which case the dipole coupling theory should be quite accurate), the power dissipated will not depend very strongly on Б. One can also find the power dissipated by simply taking it to be $(1-|R|^2)\,cHL_w|D|^2/4\pi Z$. If Б =0, this power dissipated is zero because in this case, $|R|^2 = 1$. Thus this expression for the power dissipated is a strong, increasing function of Б. At some value of Б they will be equal; this is then the value to use in this approximation and it specifies the excitation of the cavity for the case of the input power coupled to the coaxial waveguide through a waveguide T.

Acknowledgment: The author would like to thank Richard Fischer of NRL, Alan McCurdy of USC and Ronald Vernon of U. Wis. for a number of useful discussions.

References

1. J.D. Jackson, *Classical Electromagnetic Theory*, second edition, Chapters 8 and 9, J. Wiley and sons, New York, 1975

2. Robert E. Collin, *Foundation for Microwave Engineering*, second edition, Chapter 7, McGraw Hill, 1992

3. J. Gao, *Analytic Formulae for the Coupling Coefficients β between a Waveguide and a Traveling Wave Structure*, IEEE Particle Accelerator Conference, 1995

4. F. Sporleder and H.G. Unger, *Waveguide Tapers, Transitions and Couplers*, p52, Peter Peregrinis, 1979

5. Collin, op. Cit. Chapter 4

6. Samuel Liao, Microwave Circuits, p 146-156, Prentice Hall